

Graded High Field Nb₃Sn Dipole Magnets

S. Caspi, P. Ferracin and S. Gourlay

Abstract— Dipole magnets with fields beyond 16T will require superconducting coils that are at least 40 mm thick, an applied pres-stress around 150 MPa and a protection scheme for stored energy in the range of 1-2 MJ/m. The coil size will have a direct impact on the overall magnet cost and the stored energy will raise new questions on protection. To reduce coil size and minimize risk, the coil may have to be graded. Grading is achieved by splitting the coil into several layers with current densities that match the short sample field in each layer. Grading, especially at high fields, can be effective; however it will also significantly raise the stress. In this paper we report on the results of a study on the coil size and field relation to that of the stress and stored energy. We then extend the results to graded coils and attempt to address high stress issues and ways to reduce it.

Index Terms—Grading, Superconducting magnets, Stress, Stored energy.

I. INTRODUCTION

When the LHC pushed NbTi conductor closer to its high-field limit, the 56 mm bore dipoles reach 9.7 T at 1.9 K, and a stored energy of 334 kJ/m (per bore) [1]. Replacing the cable in that magnet with identical size Nb₃Sn conductor (31.3 mm of overall coil thickness) would raise the field to 15.3 T, the stored energy to 900 kJ/m, and the Lorentz stress from 88 MPa to 220 MPa. If we farther grade the coil (20.3 mm layer 1 and 11.0 mm layer 2) the central field will increase to 16.2 T, the Lorentz stress in layers 1 and 2 will increase to 202 MPa and 377 MPa respectively, and the stored energy will increase to 1.14 MJ/m. (we assume a Nb₃Sn wire carrying 3000 A/mm² at 12 T, 4.2 K in the non-copper with a 50% reduced engineering current density).

This comparison between NbTi and Nb₃Sn conductors points out both the promise and the challenge for high field magnets. Accelerator magnets can not be treated as a “one-of-a-kind” magnet; their cost must be at a minimum and the reliability high. Pushing the limits on high field Nb₃Sn magnets requires the best superconductor, a reasonably small size magnet, and a compact structure. In this paper we address the relations between field, coil size, bore diameter, stress and stored energy, and we then extend them to graded coils pointing out areas where we presently meet the challenge and areas that will require further R&D.

Manuscript received September 15, 2005. This work was supported under contract DE-AC02-05CH11231 by the Director, Office of Energy Research, Office of High Energy Physics, U.S. Department of Energy.

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II. DIPOLE MODEL

A. Model Description

To study the limits of superconducting dipole magnets, we formulated a simple but realistic model based on several assumptions: 1) the bore is round and the coil is a thick cylinder (Fig 1); 2) the engineering current density is equal to $J_e \cos \theta$ and the field is therefore a pure dipole; 3) the field magnitude in the bore and along the conductor inner surface is identical; 4) the short-sample current density in the superconductor J_{ss} and the engineering current density J_{e-ss} (obtained by averaging J_{ss} over copper, insulation, and voids) are a function of field B and temperature T ; 5) the coil is initially not graded (section II) followed by graded coils (section III); 6) there is no ferromagnetic material nearby.

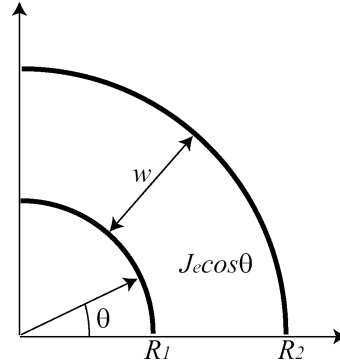


Fig. 1. A pure dipole with a $J_e \cos \theta$ current density.

We summarized our previous work on none-graded coils [2]. A simple relation exists between the central dipole field B_0 , the engineering current density J_e , and the coil thickness w [3]:

$$B_0 = \frac{\mu_0 J_e}{2} w.$$

If we extend the engineering current density J_e up to the short-sample current density limit J_{e-ss} , the field will reach its corresponding short-sample value B_{ss} . By applying Summer's empirical short-sample relation [4] to the above equation, we obtain

$$w_{ss}(B_{ss}, T) = \frac{2B_{ss}}{\mu_0 J_{e-ss}(B_{ss}, T)},$$

where the coil thickness is expressed as a function of short-sample field and temperature only. We note and emphasize that the relation is *independent* of bore diameter.

The parametric analysis assumes the best commercially available superconductor (Nb₃Sn with 3000 A/mm² at 12 T and 4.2 K) and a Rutherford cable with 50 % non-copper, a 12 % void fraction and an 11 % insulation fraction (Fig. 2).

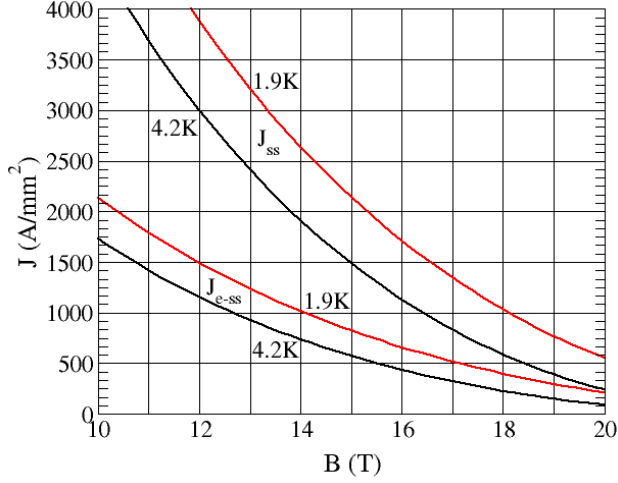


Fig. 2. Short-sample J_{ss} and engineering current density J_{e-ss} of Nb₃Sn superconductor.

B. Parametric dependencies

1) Coil Thickness

The coil thickness w_{ss} at short-sample is plotted in Fig. 3 for two different operating temperatures. We note that, at 1.9 K, a 7 mm thick coil is sufficient to generate a 10 T field, and a 100 mm thick coil will be needed for a 19 T field. Operating at 1.9 K requires significantly less conductor than at 4.2 K, an advantage that becomes even more significant as the field approaches 20 T.

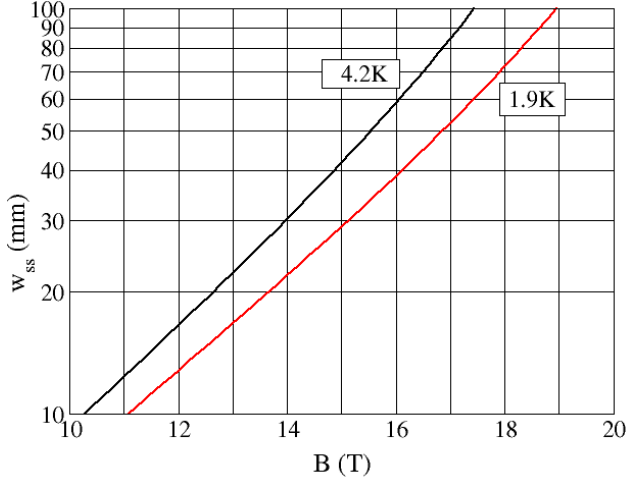


Fig. 3. Coil thickness of Nb₃Sn dipole magnets at short-sample.

2) Bore Diameter and Cost

Since the field depends on the coil thickness and not on the bore diameter, we may claim that a dipole with a zero bore diameter has the same field as a coil with any bore diameter, as long as the coil thickness is constant. This gives us the opportunity to separate the cost of the field from the cost of the bore.

The cost of the coil is proportional to its area, which is given by $\pi w^2 + 2\pi w R_l$ (where w is the coil thickness, and R_l is the bore radius). We associate the first term with the area of a no bore coil, and the second term with an additional area representing the contribution of a bore. Accordingly, the cost of the field is proportional to w^2 , but the additional cost of the bore is linearly proportional to the bore diameter. For example at high fields (18 T), doubling the bore diameter from 25 mm to 50 mm increases the amount of conductor by only 26 %. We conclude that at very high fields, the effect of the bore diameter on the overall cost of the conductor is minor (grading will be addressed in the following section).

3) Lorentz Stress

The azimuthal Lorentz stress in a $\cos\theta$ dipole is the integrated azimuthal Lorentz force with respect to θ (no shear) [5]. Expressed as

$$\sigma_\theta = \frac{\mu_0 J_e^2}{2} \frac{r}{2} \left(R_2 - \frac{R_1^3}{3r^2} - \frac{2}{3} r \right) \cos^2 \theta,$$

the stress exhibits a maximum along the coil mid-plane ($\theta = 0$), at a radius nearly two thirds of the coil thickness (setting $\partial\sigma_\theta/\partial r = 0$ and solving the cubic relation in r).

If we apply the short-sample field, coil thickness w_{ss} , and a given bore diameter Φ to the above equation, we arrive at the maximum short-sample stress $\sigma_{\theta-max-ss}$ (Fig. 4).

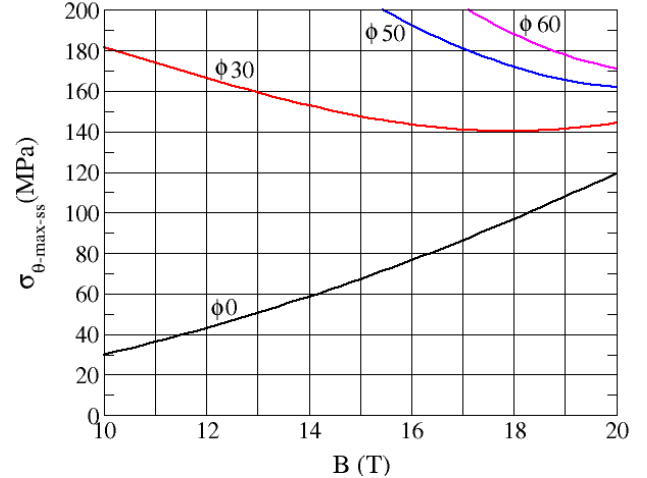


Fig. 4. Maximum azimuthal Lorentz stress as a function of the short-sample field at 1.9 K, and the bore diameter in mm.

The zero bore solution is a monotonic increasing function of the field and is at the minimum for any bore diameter at that field. Surprisingly, for certain conditions the maximum stress decreases as the short-sample field increases. As the field increases, so do coil thickness w_{ss} and Lorentz force; however, the rate of increased coil thickness is greater than the corresponding Lorentz force, thereby reducing the stress. At high fields the coil thickness dominates and the stress asymptotically approaches that for a zero bore solution, equal to $\sigma_{\theta-max-ss} = (3B_{ss}^2)/(8\mu_0)$.

4) Stored Energy

The stored energy E of a dipole increases quadratically with field B , bore radius R_l , and coil thickness w :

$$E = \frac{\pi B^2}{3\mu_0} \frac{w^2}{2} \left[1 + 6 \left(\frac{R_l}{w} \right)^2 + 4 \left(\frac{R_l}{w} \right) \right].$$

Fig. 5 is a log plot of the short-sample stored energy E_{ss} for a number of different bore diameters, including a zero bore.

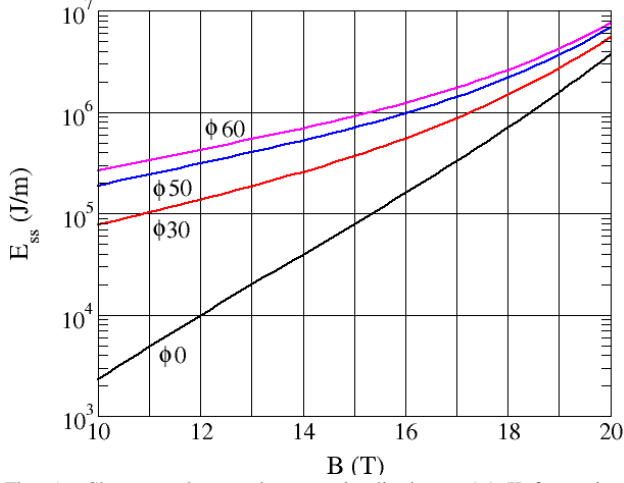


Fig. 5. Short-sample stored energy in dipoles at 1.9 K for various bore diameters.

As for the stress, we can associate the energy of a zero bore diameter with the term outside the square brackets in the formula above, and the terms within the bracket as an additional bore contribution. At high fields, when the coil thickness w increases, the additional contribution of bore diameter to the stored energy becomes less effective and the stored energy asymptotically approaches that of a zero bore.

If we focus on the curves representing a bore diameter closer to the LHC dipole (50-60 mm), we notice that at 15 T the stored energy is close to 1 MJ/m reaching 7 MJ/m at 20 T.

III. GRADED COILS

A. Model Description

Grading the coil can effectively reduce overall size while the field remains the same. Grading takes advantage of the drop in field within the coil in order to raise the current density in several discrete outer layers, thus matching that drop. That way the superconductor can reach its short-sample simultaneously throughout the cross-section, thereby reducing the overall size.

1) Coil Thickness

Grading significantly impacts the coil thickness especially at high field: for example, at 16 T a two layer graded coil reduces its thickness by 23%. That number increases to 32% at 18 T and 42% at 20 T (Fig. 6). Increasing the number of graded coils from 2 to 4 reduces the overall coil thickness by an additional 10%. Fig. 7-9 show the relation between coil thickness, current density and field at the conductor as a function of the dipole field of a 50 mm bore magnet.

2) Bore Diameter

Whereas in a non-graded magnet the coil thickness does not depend on the bore diameter that is no longer true for graded coils. The overall coil thickness of a graded coil increases by

5% for an increase of bore diameter from a 30 mm to 50 mm.

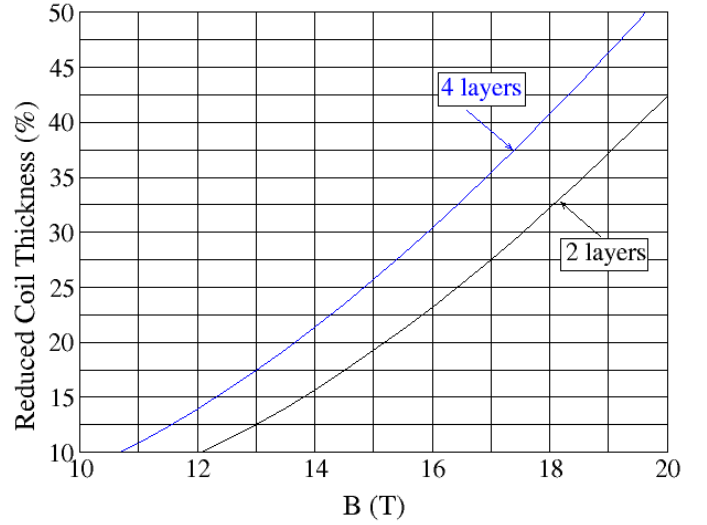


Fig. 6. Reduction in coil thickness of a 2 layer and a 4 layer graded magnet at short-sample (1.9 K).

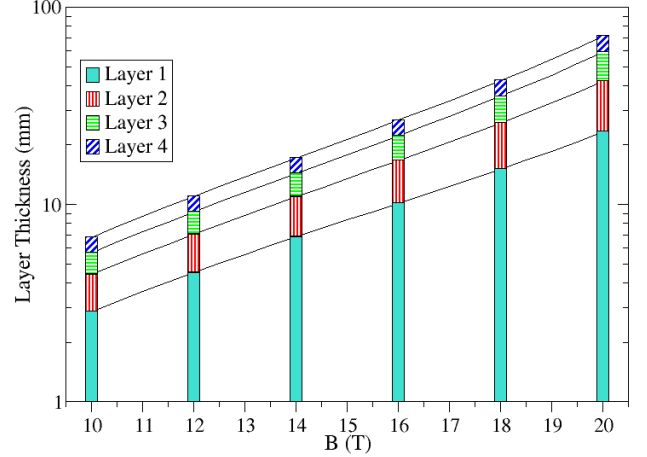


Fig. 7. Incremental layer thickness of a 4 layer graded coil 50 mm bore magnet (1.9 K).

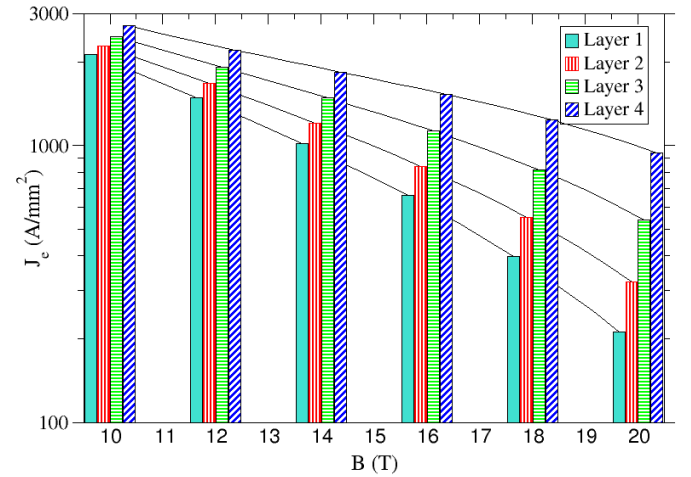


Fig. 8. Engineering current density of a 4 layer graded 50 mm bore magnet (1.9 K).

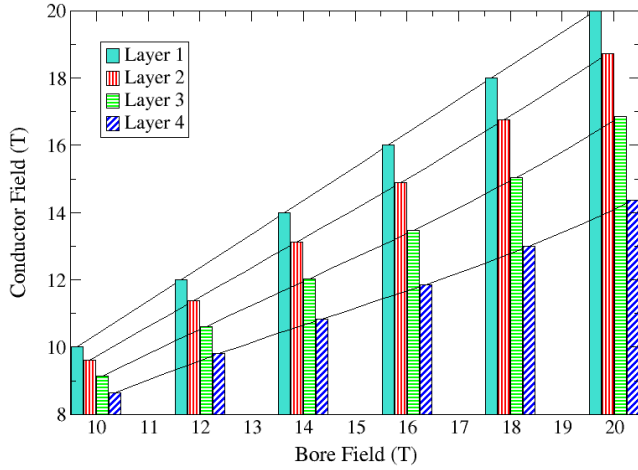


Fig. 9. Field in the conductor of each layer of a 4 layer graded magnet (50 mm bore, 1.9 K).

3) Lorentz Stress

Grading reduces the coil thickness at the expense of an increase in conductor stress. At 18 T (1.9 K) grading will reduce the coil thickness from 72 mm to 49 mm, but, in a 56 mm bore diameter, the maximum stress will rise from 170 MPa to an unacceptable level of 380 MPa (Fig. 10,11). Since reducing the coil thickness has a major impact on reducing the magnet cost, we are left with an important R&D issue on how to bring down the stresses. Reducing the stress through stress management should be confined to the second layer of a two layer graded magnet thereby maintaining overall high engineering current density.

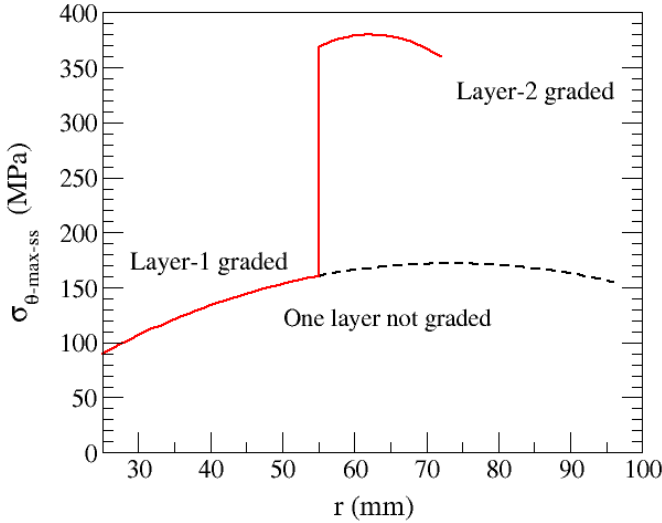


Fig. 10. Grading an 18 T dipole reduces the coil thickness but raises the stress (56 mm bore, $T = 1.9$ K).

The formulas for the mid-plane Lorentz stress in a two layers graded magnet with current densities J_{e1} and J_{e2} are written below,

$$\sigma_{\theta 1} = \frac{\mu_0 J_{e1}}{2} \frac{r}{2} \left[J_{e1} \left(R_2 - \frac{R_1^3}{3r^2} - \frac{2}{3}r \right) + J_{e2} (R_3 - R_2) \right]$$

$$\sigma_{\theta 2} = \frac{\mu_0 J_{e2}}{2} \frac{r}{2} \left[J_{e2} \left(R_3 - \frac{R_2^3}{3r^2} - \frac{2}{3}r \right) + J_{e1} \frac{R_2^3 - R_1^3}{3r^2} \right]$$

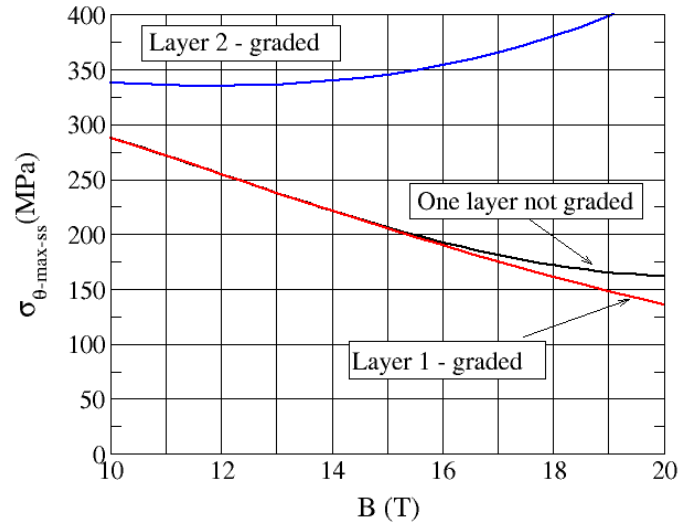


Fig. 11. Maximum stress in a graded (2 layers) and none graded magnet (50 mm bore, 1.9 K).

4) Stored Energy

Grading has only a small affect on the stored energy. At 18 T the stored energy of a 50 mm bore magnet is 1.9 MJ/m a decrease of 10 % from the 2.17 MJ/m none graded magnet (Fig. 5).

IV. CONCLUSION

The following conclusions, regarding minimum coil size, coil stress, and magnet stored energy, can be drawn: pushing dipole fields to 18 T at 1.9K in a 50 mm bore would require a 72 mm thick none-graded coil, the Lorentz stress would be 170 MPa and the stored energy 1.8 MJ/m.

Grading the same coil into two layers will reduce the overall coil thickness to 49 mm, reduce the Lorentz stress in layer one (31 mm thick) to 160 MPa, increase the Lorentz stress in layer 2 (18 mm thick) to 380 MPa and reduce the stored energy to 1.9 MJ/m.

At very high fields, the effect of bore size on conductor amount, peak stress, and the stored energy is small.

Grading the coils reduces the overall thickness at the expense of higher stress. Assuming we are at coil stress limit (150-200 MPa), new designs will have to be developed that intercept the Lorentz forces. However we need to be aware that stress management reduces not just the stress, but the coils current density as well. Stress management of the outer layer only is a good way to keep the current density high and reduce the stress.

V. REFERENCES

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